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Designing for Six-Sigma Quality with Robust Optimization Using CAE

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ABSTRACT

Although great advances have been made over the last two decades in the automotive structural design process, tradition and experience guide many design choices even today. The need for innovative tools is stronger now more than ever before as the design engineer is confronted with more complex, often contradictory design requirements such as cost, weight, performance, safety, time to market, life cycle, aesthetics, environmental impact, changes in the industry's business models, etc.

The ever-increasing use of optimization tools in engineering design generates solutions that are very close to the limits of the design constraints, hardly allowing for tolerances to compensate for uncontrollable factors such as manufacturing imperfections. Optimum designs developed without consideration of uncertainty can lead to non-robust designs. Reliability-Based Design Optimization (RBDO) methodologies not only provide improved designs but also a confidence range for simulation-based optimum designs.

In this research effort, a six-sigma robust design formulation is presented along with an example that demonstrates the advantage of robust versus deterministic optimization.

INTRODUCTION

The need for innovative tools is apparent now more than ever as more complex design requirements are surfacing such as cost, performance, safety, quality, time to market, short life cycle, environmental impacts, wow aesthetics (creating a passion for the product: "I've got to have it") and major changes in industries' business models. Moreover, the automotive industry's cycle development time from concept to production is being compressed significantly. Some of the changes in the automotive industry's business model include: vehicle designs are tailored to focused markets; vehicles are being manufactured more on a global scale; and vehicles are designed increasingly through multiple engineering sites around the world.

Quality issues are addressed early in the design cycle with robust design is a methodologies. The goal of robust design is to deliver customer expectations at affordable cost regardless of customer usage, degradation over product life and variation in manufacturing, suppliers, distribution, delivery and installation. Since randomness and scatter is a part of reality everywhere, probabilistic design techniques are necessary to engineer quality into designs. Traditional deterministic approaches account for uncertainties through the use of empirical safety factors. The safety factors are derived based on past experience; they do not guarantee satisfactory performance and do not provide sufficient information to achieve optimal use of available resources. The probabilistic design process has not been widely used because it has been intimidating and tedious due to its complexity. In recent years, FEA codes have introduced integrated probabilistic systems (e.g. ANSYS PDS) that make probabilistic analysis setup simple if the control and the noise parameters are identifiable [Ref. 1]. Control parameters are those factors that the designer can control, such as geometric design variables, material selection, design configurations and manufacturing process settings [Ref. 2]. Noise parameters on the other hand are factors that are beyond the control of the designer, such as material property variability, manufacturing process limitations, environment temperature, humidity, component degradation with time etc.

This paper describes a technique to perform probabilistic analysis, reliability based optimization and robust optimization.

FORMULATIONS FOR DETERMINISTIC, RELIABILITY AND ROBUST OPTIMIZATION

Deterministic Optimization

The ever-increasing use of optimization tools in engineering designs generates designs that are on the design constraint limits leaving very little or no room for tolerances in modeling uncertainties and manufacturing imperfections [Ref. 3]. Optimum designs developed without consideration of uncertainty could possibly lead to unreliable designs. In addition to the active constraint problems, optimum designs may be sensitive to design parameters such that small changes in the design variables may lead to a significant loss of performance. A possible formulation of deterministic optimization from automotive crashworthiness (with fictitious numbers) can be:

Minimize Weight,

subject to:

$$\text{B-Pillar}_{\text{Velocity}} \le 10\ \text{mm/s}$$
$$\text{Abdomen}_{\text{Load}} \le 1.0\ \text{KN}$$
$$\text{Rib}_{\text{Displacement}} \le 30\ \text{mm}$$
$$\text{Symphysis}_{\text{Load}} \le 5.0\ \text{KN}$$

Thickness Design Variables:

$$t^i_{min} \le t^i \le t^i_{max}$$

Reliability Index Approach

For reliability based design, a **performance function** can be defined as $\boxed{G = R - S}$ where R and S are statistically independent and normally distributed random variables of the resistance and load measurements of the structure. Typically, R can be the yield stress and S the maximum Von Mises stress. The **G** function is also called **limit state function** or **failure function.** The curve G = 0 divides the design space into two regions, the safe region when G > 0 and unsafe region when G < 0. Because we consider R and S to have variation, G will also exhibit variation. The ratio β of the mean value of the G function (μ_G) and the standard deviation of the G function (σ_G) is defined as safety index or reliability index. If Φ is the cumulative distribution function and G has a normal distribution, then:

$$\beta = -\Phi\,(1 - \text{Reliability}) = \mu_G / \sigma_G$$

A possible formulation can be:

Maximize β,

subject to:

$$\text{Weight} < \text{Target-Weight}$$
$$DV^i_{min} < DV^i < DV^i_{max}$$

Another formulation using the reliability index can be:

Minimize Weight,

subject to:

$$\beta > \text{Target } \beta$$
$$DV^i_{min} < DV^i < DV^i_{max}$$

A typical target value β is 3, which corresponds to a probability of failure of 0.00135. However, it has been observed that the Reliability Index Approach exhibits very slow convergence or even divergence for some problems.

Reliability Optimization Approach

Significant research effort has been devoted to making reliability based structural design optimization practical [Ref. 4, 5, 6]. Reliability based optimization requires two major steps. First, we need to identify the random variables and qualify the causes of variation. Typically the distribution type and the necessary values that describe the distribution function of the random variables (i.e. mean and standard deviation for a normal distribution) need to be found. Second, we need to select the desired **reliability level** (say, 90%) and reformulate the deterministic constraints as probabilistic constraints. For example a 90% reliability goal may be expressed as: the probability of failure P_f of the Abdomen Force to be greater than 1 KN must be less than 10%. A possible formulation of reliability-based optimization with a reliability goal of 95% from automotive crashworthiness (with fictitious numbers) can be:

Minimize Weight,

subject to:

$$P_{\text{failure}}\,[\,\text{B-Pillar}_{\text{Velocity}} > 10\ \text{mm/s}\,] \le 5\%$$
$$P_{\text{failure}}\,[\,\text{Abdomen}_{\text{Load}} > 1.0\ \text{KN}\,] \le 5\%$$
$$P_{\text{failure}}\,[\,\text{Rib}_{\text{Displacement}} > 30\ \text{mm}\,] \le 5\%$$
$$P_{\text{failure}}\,[\,\text{Symphysis}_{\text{Load}} > 5.0\ \text{KN}\,] \le 5\%$$

Design Variables:

Mean values of various design parameters,

$$\mu t^i_{min} \le \mu t^i \le \mu t^i_{max}$$

Random Variables with known or assumed variation:

Thickness
Yield Stress
Barrier Height
Impact Position

The reliability based optimization approach accounts for variation and generates designs that meet a given level of reliability and usually move optimum solutions away from the constraints.

Robust Optimization Approach

The robust design optimization approach not only shifts the performance mean to the target value but also reduces the product's performance variability, achieving **Six-sigma level** robustness on the key product performance characteristics with respect to the quantified variation [Ref. 7, 8]. A possible formulation of a 6-sigma level robust design optimization approach from automotive crashworthiness (with fictitious numbers) can be:

Minimize Weight,

subject to:

μ [B-Pillar $_{\text{Velocity}}$] + 6 σ[B-Pillar $_{\text{Velocity}}$] $\leq$ 10 mm/s
μ [Abdomen $_{\text{Load}}$] + 6 σ [Abdomen $_{\text{Load}}$] $\leq$ 1.0 KN
μ [Rib $_{\text{Displacement}}$] + 6 σ [Rib $_{\text{Displacement}}$] $\leq$ 30 mm
μ [Symphysis $_{\text{Load}}$] + 6 σ [Symphysis $_{\text{Load}}$] $\leq$ 5.0 KN

Design Variables: Mean values or standard deviation of design parameters,

$$\mu t^{i}_{min} \leq \mu t^{i} \leq \mu t^{i}_{max}$$
$$\sigma t^{i}_{min} \leq \sigma t^{i} \leq \sigma t^{i}_{max}$$

Random Variables with known or assumed variation:

Thickness
Yield Stress
Barrier Height
Impact Position

The Parametric Deterministic FEA Model

In this research effort, the probabilistic modeling of manufacturing (thickness) and loading variations for the radiator support of a SUV was considered. A parametric finite element model of the radiator support was developed considering the thickness t and the load P as parameters. It was assumed that the material is isotropic, linear elastic and the behavior is within small deflection linear theory limits. The Modulus of Elasticity, E is assumed to be 200000 MPa and Poisson's ratio, ν = 0.25. Several load cases were evaluated and the torsional load case was found to control the maximum Von Mises stress. The distribution of the magnitude of the displacement vector for this load case is shown in Figure 1. The corresponding Von Mises stress distribution is shown in Figure 2.

For any combination of the input parameters (t and P) the solution to the parametric model can compute the maximum Von Mises stress $\boldsymbol{\sigma_{eq}}$. We define a performance function G as the difference between yield stress and the maximum computed Von Mises stress or

$$\mathbf{G = \sigma_y - \sigma_{eq}}$$

If G remains positive at all time, we have a safe design.

The performance function G is considered the output variable and is a function of the input variables **t** and **P**.

The data flow for the parametric FE model is shown in the blue box in Figure 3. For a given geometry and a set of values of the input parameters, the deterministic parametric FE model can predict the value of the performance function G.

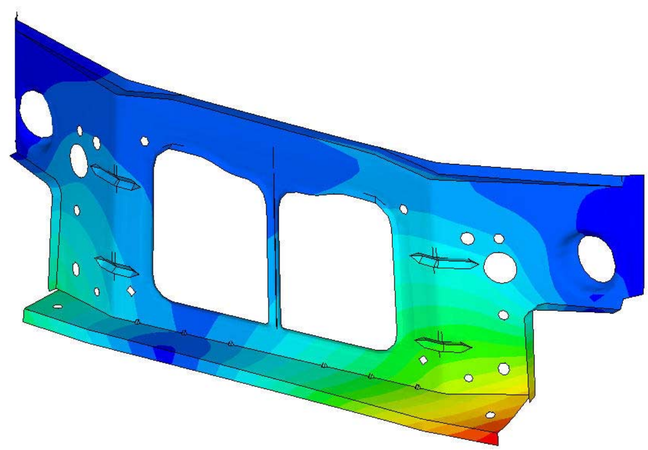

Figure 1 Displacement Distribution Torsional Load Case

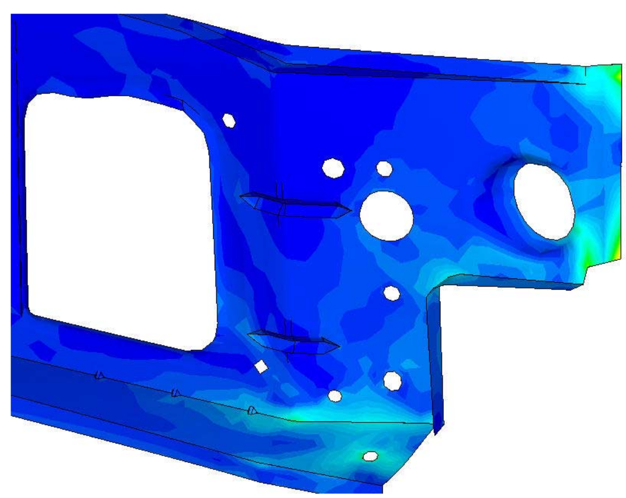

Figure 2 Von Mises Distribution for Torsional Load Case

The Probabilistic FEA Model

Uncertainty in the input parameters of the FEA model can be introduced by assuming certain randomness in the input parameters. In this study, it was assumed that a Gaussian distribution with mean value μ_t and standard variation σ_t characterizes the variation in thickness t. The mean value μ_t was considered as a *controllable* parameter and it was declared an ***optimization design variable***. The standard variation σ_t was considered as 3% of the mean value μ_t. The applied load P was considered as uncontrollable or a noise parameter that exhibits a lognormal distribution with a mean value μ_P = 44 N and standard variation σ_P = 8.8 N. Figure 4 shows the Probability Distribution of the input variable P.

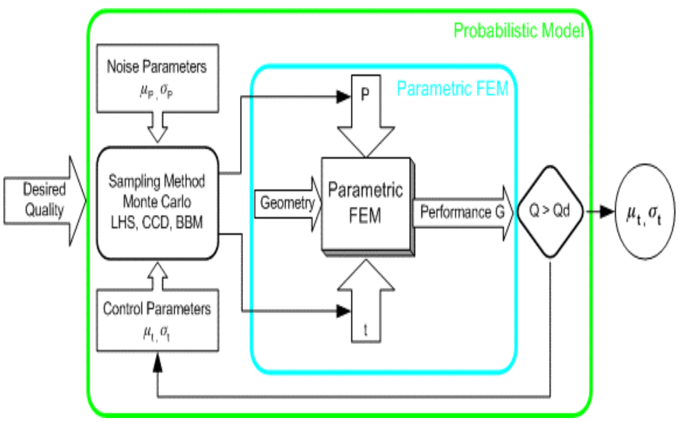

Figure 3 Data Flow for Probabilistic & Parametric FEM Models

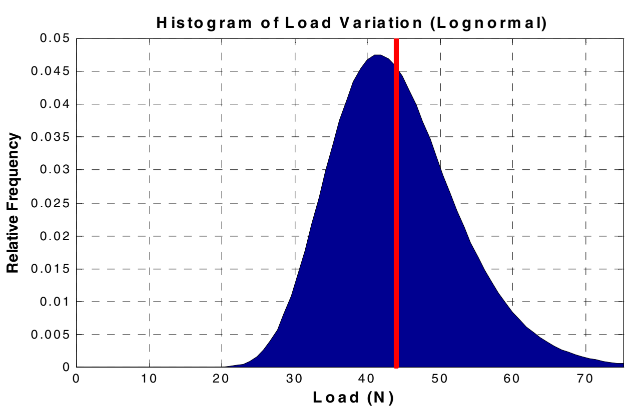

Figure 4 Probability Distribution of the input variable P

The top part of Figure 5 shows the Probability Distribution of the input variable t with mean value μ_t = 1.10 mm and standard variation σ_t = 0.033 mm. The lower part of Figure 5 shows the Probability Distribution of the input variable t with mean value μ_t = 0.88 mm and standard variation σ_t = 0.0264 mm.

Monte Carlo and the Central Composite Design response surface sampling techniques were implemented in determining the response distribution of the output variable G for various values of the mean thickness value μ_t. Figure 3 shows the data flow of the probabilistic model. A sampling method is selected that combines both noise and control random variables to produce a set of deterministic runs, using the parametric FE model. Post processing of these runs determines the probability distribution of the performance function G.

Figure 6 shows the Probability Distribution for three different values of mean thickness μ_t = 0.85, 1.0 and 2.0. One may observe that for μ_t = 2.0, the entire distribution of the performance function G remains on the positive side indicating that for μ_t = 2.0 the maximum Von Mises stress does not exceed the yield shear stress. In this case the mean value of the performance function μ_G = 163.47 MPa, the standard deviation σ_G = 7.56 MPa. and the probability that the performance function G is less than zero is 0 %, P[G<0] = 0%.

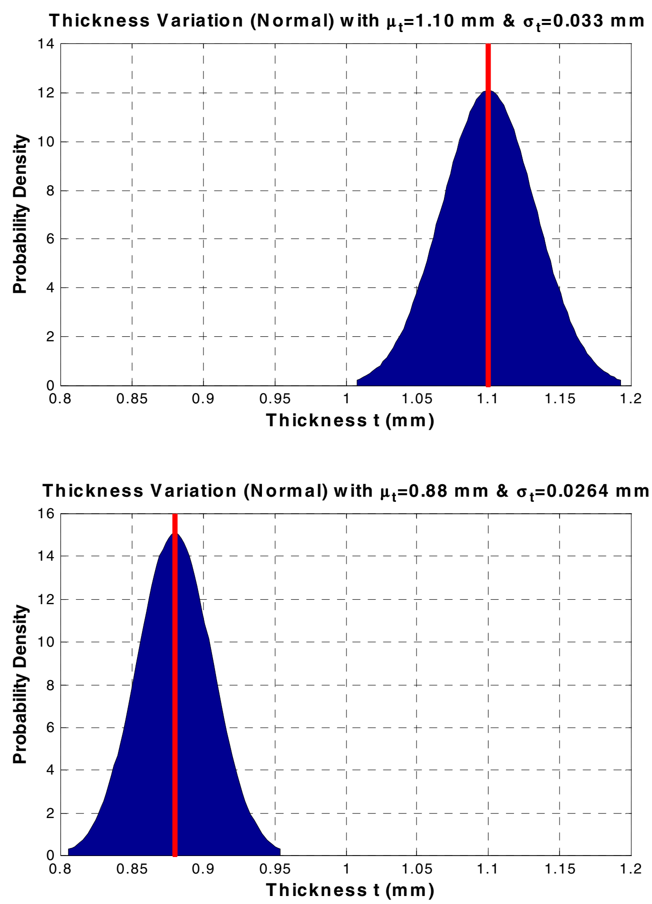

Figure 5 Probability Distribution of the input variable t with μ_t = 1.10 mm and σ_t = 0.033 mm (top) and μ_t = 0.88 mm and σ_t = 0.0264 mm (bottom)

For μ_t = 1.0, one may observe that part of the distribution of the performance function G remains on the positive side indicating that for μ_t = 1.0 the maximum Von Mises stress some times exceeds the yield shear stress. The area to the left of the zero (red line) indicates the probability of failure. In this case, the mean value of the performance function μ_G = 64.676 MPa, the standard deviation σ_G = 27.90 MPa. and the probability that the performance function G is less than zero is 2.3 %, P[G<0] = 2.3%.

Similarly for μ_t = 0.88, a smaller part of the distribution of the performance function G remains on the positive side indicating that for μ_t = 0.88 the maximum Von Mises stress sometimes exceeds the yield shear stress. In this case the mean value of the performance function, μ_G = 27.747 MPa, the standard deviation σ_G = 35.77 MPa. and the probability that the performance function G is less than zero is 20.2 %, P[G<0] = 20.2%.

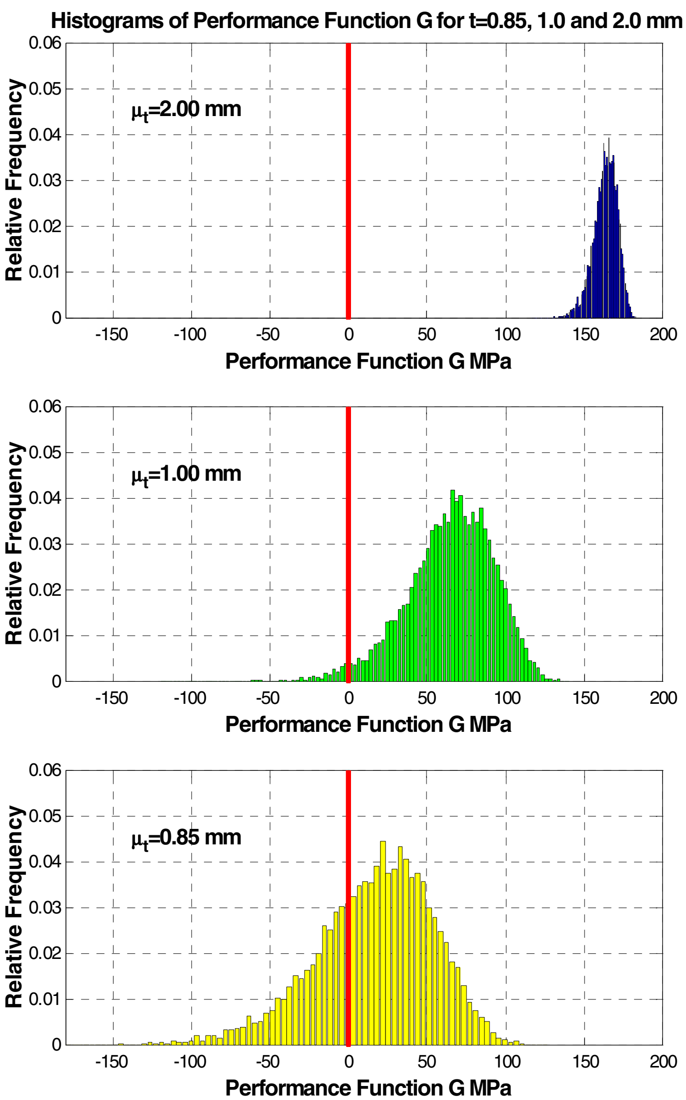

Figure 6 Distributions of Performance Function G for various values of μ_t

Results of Reliability Based Analysis

For various values of the standard deviation μ_t, the probabilistic FEA model can not only predict the mean value of the performance function μ_G, and the standard deviation of the performance function σ_G, but also the probability that the performance function is less than zero P[G < 0]. Table 1 shows the mean, the standard deviation of performance function G and probabilities of failure for various values of the mean thickness value μ_t. Figure 7 shows a plot of the probability that the performance function is less than zero versus the mean thickness value μ_t. One may observe that for values of μ_t = 0.9633, 0.9281 and 0.8995 mm, the probability of failure is 5%, 10% and 15% respectively. This graph can be used as a design guide to select the required mean thickness value μ_t to achieve the desired reliability level. For example, if the desired reliability level is 95% the minimum mean thickness value μ_t should be 0.9633 mm.

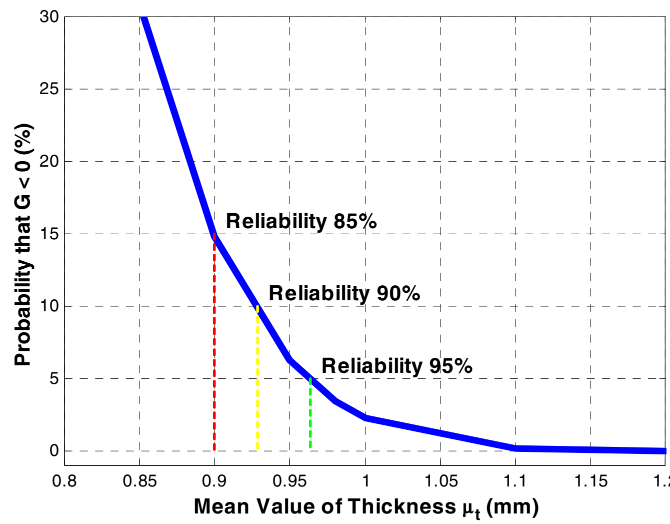

Figure 7 Probability that the Performance Function is less than zero vs. the Mean Thickness Value μ_t

Results Of Robust Analysis

The robust design optimization approach not only shifts the performance mean to the target value but also reduces a product's performance variability, achieving the desired **sigma level** robustness on the key product performance characteristics with respect to the quantified variation.

In this study a typical formulation of an n-sigma level robust design approach can be stated as:

Find the value of the minimum mean thickness value μ_t in order to achieve positive values of the expression

$$\mu_G - n\,\sigma_G > 0$$

Figure 8 shows a sensitivity plot of six curves for 1-σ through 6-σ versus the mean thickness value μ_t. For a 3-σ quality level, the mean thickness value μ_t should be greater than μ_t = 1.0536 mm. For a 6-σ quality level, the mean thickness value μ_t should be greater than μ_t = 1.2558 mm.

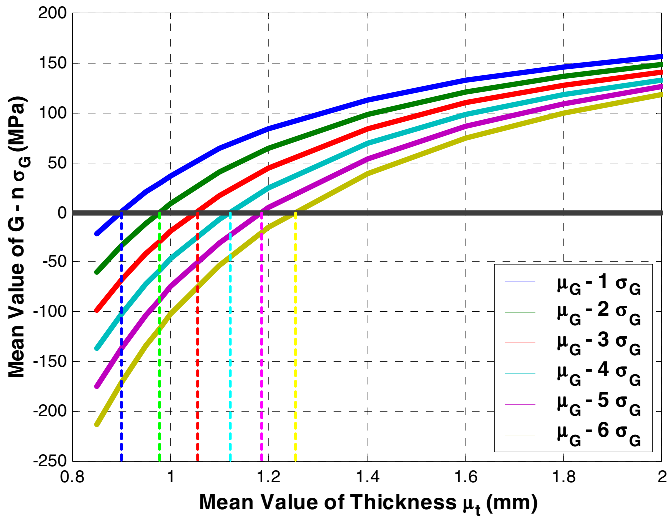

Figure 8 1-σ through 6-σ quality level curves vs. the standard deviation of mean thickness value μ_t

CONCLUSIONS

The example presented demonstrates the advantage of using an automated probabilistic design process that enables engineers to identify better designs that meet the performance objectives and are less sensitive to manufacturing variations.

For a given sigma quality level (i.e. six-sigma) the mean thickness value μ_t can be determined using the design process described. The results summary is also shown in Table 2.

For a given reliability goal (i.e. 95%) the mean thickness value μ_t can be determined using the design process described. The results summary is shown in Table 2.

By incorporating the physical scatter into the model, the risk of failing legal or consumer tests can be minimized.

μ_t (mm)	μ_G (Mpa)	σ_G (Mpa)	P[G<0] (%)	Min G (Mpa)	Max G (Mpa)
Mean t Value	Mean G Value	Standard Deviation of G	Probability that G < 0	Minimum G Value	Maximum G Value
0.85	16.118	38.092	30.78	-241.1	125.3
0.88	27.747	35.774	20.23	-195.0	126.5
0.90	34.891	34.301	14.83	-146.5	132.9
0.95	50.919	30.855	6.25	-115.5	132.6
0.98	59.384	29.200	3.427	-83.97	140.0
1.00	64.676	27.901	2.31	-118.5	139.0
1.10	86.906	23.477	0.180	-50.56	149.6
1.20	104.03	19.862	0.037	-152.1	156.4
1.40	128.25	14.860	0.00	50.54	168.7
1.60	144.25	11.525	0.00	77.88	176.1
1.80	155.40	9.209	0.00	100.00	181.24
2.00	163.47	7.5757	0.00	117.2	182.8

Table 1: Mean, Standard Deviation of Performance Function G and Probabilities of Failure for various values of the Mean thickness value μ_t

Quality / Reliability	Mean Thickness, μ_t (mm)
1-σ Quality Level	0.8987
2-σ Quality Level	0.9775
3-σ Quality Level	1.0536
4-σ Quality Level	1.1222
5-σ Quality Level	1.1866
6-σ Quality Level	1.2558
85% Reliability Level	0.8995
90% Reliability Level	0.9281
95% Reliability Level	0.9633

Table 2: Mean Thickness μ_t for various quality level

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