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ABSTRACT

A design is robust when the performance targets have been achieved and the effects of variation have been minimized without eliminating the causes of the variation such as manufacturing tolerances, material properties, environmental temperature, humidity, operational wear etc. In recent years several robust design concepts have been introduced in an effort to obtain optimum designs and minimize the variation in the product characteristics [1,2]. In this study, a probabilistic design analysis was performed on a catalytic converter substrate in order to determine the required manufacturing tolerance that results in a robust design. Variation in circularity (roundness) and the ultimate shear stress of the substrate material were considered. The required manufacturing tolerance for a robust design with 1,2 and 3 sigma quality levels was determined. The same manufacturing tolerance for a reliability based design with reliability levels of 85%, 90% and 95% was also determined and compared. The methodology for implementing robust design used in this research effort is summarized in a reusable workflow diagram.

INTRODUCTION

Robust design is a methodology that addresses product quality issues early in the design cycle. The goal of robust design is to deliver customer expectations at profitable cost regardless of customer usage, degradation over product life and variation in manufacturing, suppliers, distribution, delivery and installation. Since randomness and scatter is a part of reality everywhere, probabilistic design techniques are necessary to engineer quality into designs. Traditional deterministic approaches account for uncertainties through the use of empirical safety factors. The safety factors are derived based on past experience; they do not guarantee safety or satisfactory performance and do not provide sufficient information to achieve optimal use of available resources. The probabilistic design process has not been widely used because it has been intimidating and tedious due to its complexity.

In this research effort, probabilistic modeling of manufacturing and material variations for a catalytic converter substrate was considered. Typical shapes of catalytic converter substrates are shown in Figure 1. The substrate used in this study has a cylindrical cross section and is enclosed in a cylindrical steel cover. If the substrate is not a perfect cylinder the steel cover applies a non-uniform pressure along the circumference. Assuming that the maximum diameter of the substrate is F_{max} and the minimum diameter is F_{min}, we can characterize the variation in circularity or roundness d with their difference $d = F_{max} - F_{min}$. Due to manufacturing variations d is considered a random input variable.

Figure 1 Typical shapes of catalytic converter substrates

In this study, it was assumed that a Gaussian distribution with mean value m_d and standard variation s_d characterizes the variation in circularity d.

Due to both material and manufacturing variation the ultimate shear stress t_{ult} exhibits randomness. A pure shear test was performed on several substrates to determine the ultimate shear stress variation. The mean value of ultimate shear stress is m_{tu} = 0.2868 MPa (41.6 psi), the standard deviation is s_{tu} = 0.01724 MPa (2.5 psi), the minimum value of the sample was t_{umin} = 0.255 MPa (37.0 psi) and the maximum value of the sample was t_{umax} = 0.31 MPa (45.0 psi).

The objective of this study is to identify the supplier specification max s_d (maximum standard deviation of variation in circularity d) in order to achieve a robust design of a desired sigma quality level.

The Parametric Deterministic FEA Model

A parametric finite element model of the substrate is shown in Figure 2 and was developed considering the following assumptions:

1. The material of the honeycomb ceramic substrate is isotropic, linear elastic and the behavior is within small deflection linear theory limits. The modulus of elasticity is E = 4800 MPa and Poisson's ratio is ν = 0.25
2. Plain Strain Analysis is sufficient to accurately predict the maximum shear stress.
3. There is no temperature effect on the material properties.
4. The geometry, loading and behavior are symmetric about the horizontal and vertical axes, thus a quarter symmetry model can be used.
5. The angle between the maximum and minimum diameter is 90°.
6. The geometry of the FEA model can be represented by the first quadrant of an ellipsoid with the horizontal lower edge length equal to **F/2 + d/4**, and the vertical left edge length equal to **F/2 - d/4**, where F = 105.0 mm.
7. The maximum pressure P_{max} (in MPa) is a function of δ (in mm) and can be computed by the following equation:

   $$Pmax(\delta) = a_5\delta^5 + a_4\delta^4 + a_3\delta^3 + a_2\delta^2 + a_1\delta^1 + a_0$$

   where:

   a_0 = -0.8155
   a_1 = 5.5840
   a_2 = -8.3864
   a_3 = 4.0882
   a_4 = -0.0694
   a_5 = 0.0004

   Figure 3 shows a plot of this function (the maximum pressure P_{max} versus δ) and the data points used for the curve fitting.
8. As shown in figure 2 the maximum pressure is applied at the point of maximum diameter and varies sinusoidally to zero at the point of minimum diameter:

   $$P(\theta) = P_{max} * \cos(\theta)$$

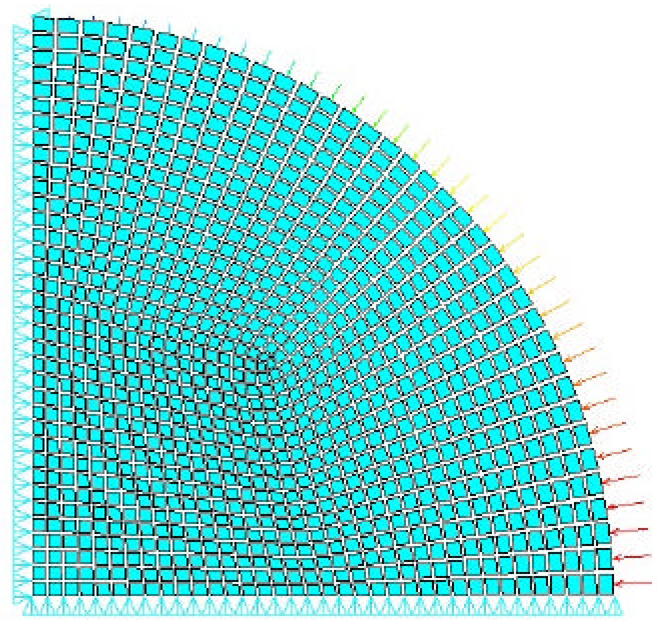

Figure 2 Finite Model and Loading of the Substrate

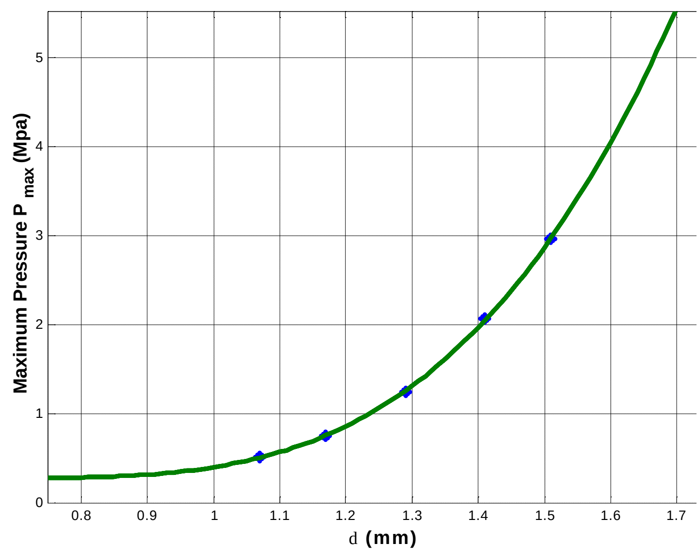

Figure 3 Maximum Pressure P_{max} versus δ

The input parameters of the model are variation in circularity d and ultimate shear stress t_{ult}. For any combination of the input parameters the solution to the parametric model can compute the maximum shear stress t_{max}. Figure 4 shows a typical distribution of the magnitude of the displacements and figure 5 shows a typical maximum shear stress distribution.

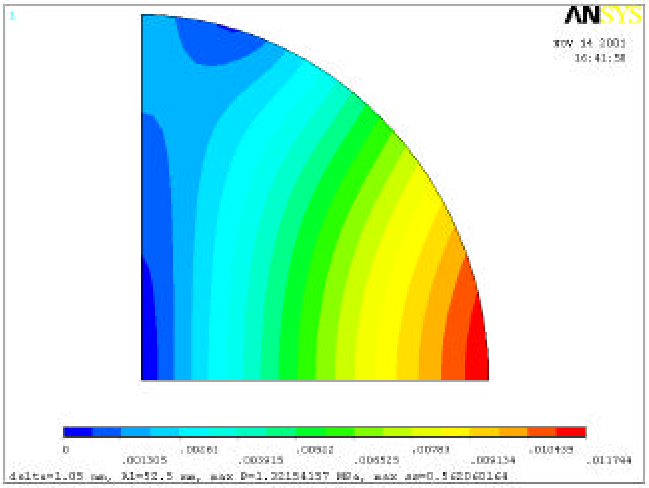

Figure 4 Typical Displacement Distribution

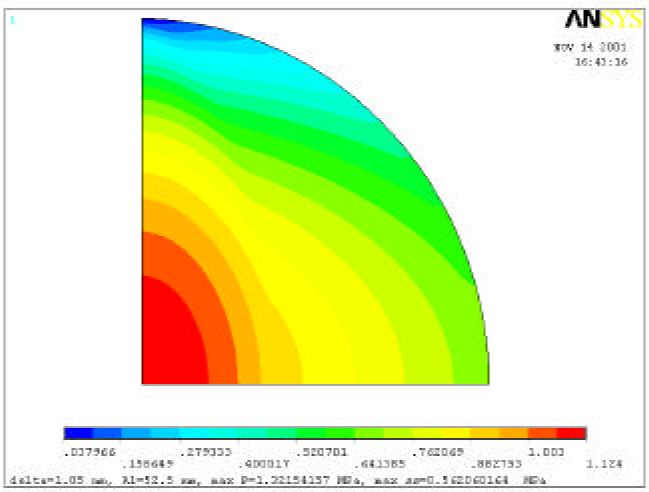

Figure 5 Typical Maximum Shear Stress Distribution

We define a performance function G as the difference between ultimate shear stress and the maximum computed stress or **G =** t_{ult} **-**t_{max}**.** If G remains positive at all times we will have a safe design. The performance function G is considered as the output variable and is a function of the input variables d and t_{ult}**.**

The Probabilistic FEA Model

Uncertainty in the input parameters of the FEA model can be introduced by assuming certain randomness in the input parameters. In this study, it was assumed that a Gaussian distribution with mean value m_d and standard variation s_d characterizes the variation in circularity d. The mean value m_d was considered uncontrollable or a noise parameter with a constant value of m_d = 1.05 mm**.**

The standard variation s_d was considered as a *controllable* parameter and it was declared as an ***optimization design variable***. For various values of the standard variation s_d one may obtain a distribution for d**.** Figure 6 shows the Probability Distribution of the input variable δ with a standard variation s_d = 0.01 mm.

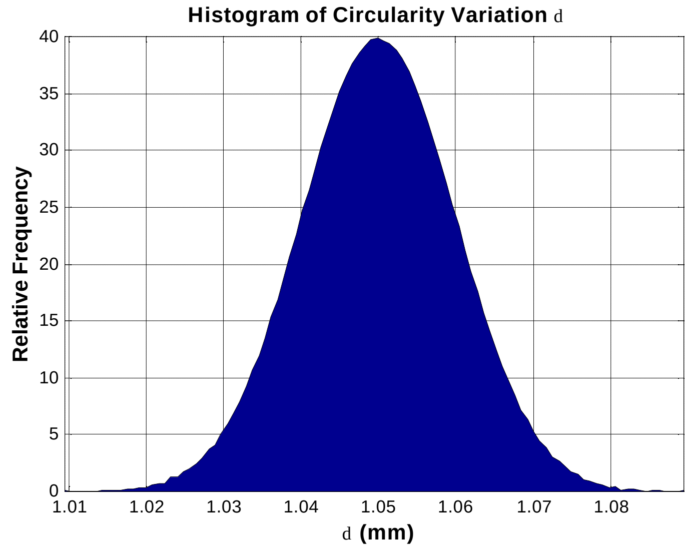

Figure 6. Probability Distribution of the Input Variable δ

It was also assumed that truncated Gaussian distribution characterizes the ultimate shear stress of the substrate material. The mean value of ultimate shear stress m_{tu} = 0.2868 MPa, the standard deviation s_{tu} = 0.01724 MPa, and the range 0.255 - 0.310 MPa were determined experimentally. Figure 7 shows the Probability Distribution of the input variable s_{tu} with these values.

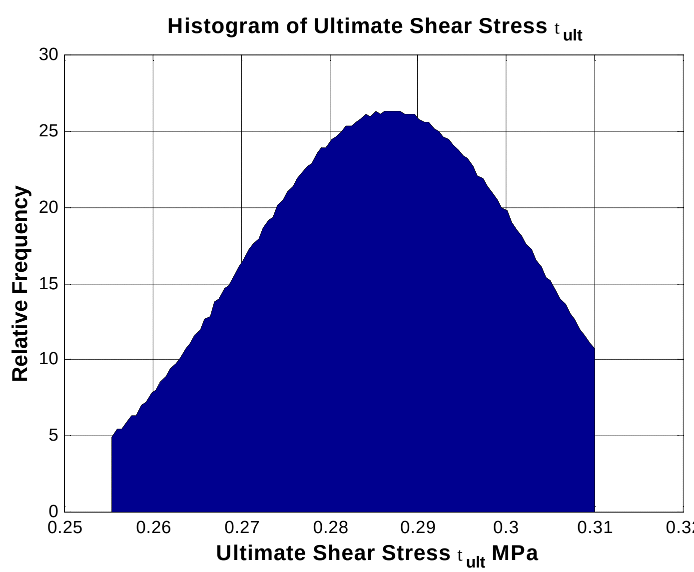

Figure 7 Probability Distribution of the Input Variable $\sigma_{\tau u}$

Since the two input variables are random the performance function G (where **G =** t_{ult} **-**t_{max}**)** exhibits randomness and is considered the output variable.

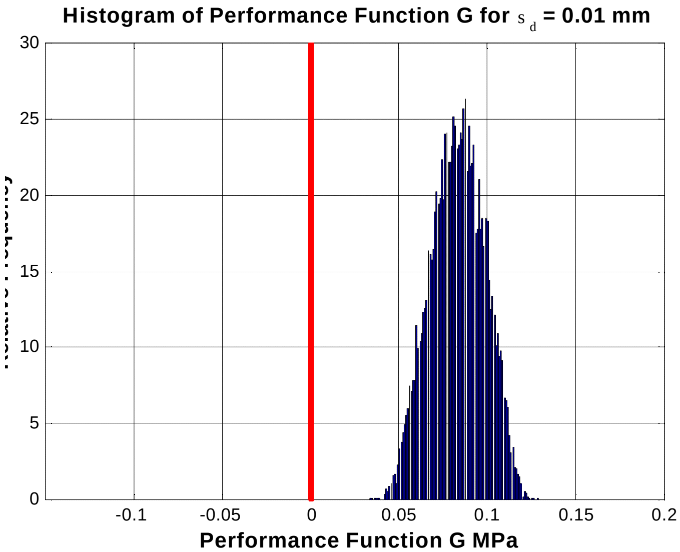

Figure 8 Probability Distribution of the Performance Function G for Standard Variation σ_δ = 0.01 mm.

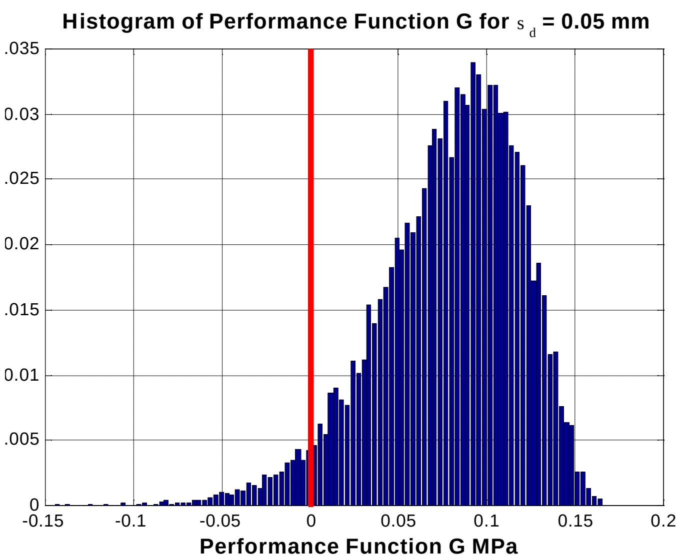

Figure 9 Probability Distribution of the Performance Function G for Standard Variation σ_δ = 0.05 mm.

Monte Carlo and the Central Composite Design response surface sampling techniques were implemented in determining the response distribution of the output variable for various values of the standard variation $s_{d.}$

The probability distribution of the performance function G for standard variation s_d = 0.01 mm is shown in Figure 8. The mean value of the performance function is m_G = 0.08312 MPa and the standard deviation is s_G = 0.01521 MPA. One may observe that the entire distribution of the performance function G remains on the positive side, indicating that for s_d = 0.01 the maximum shear stress does not exceed the ultimate shear stress. In this case the probability that the performance function G is less than zero is 0 %, P[G<0] = 0%. Figure 9 shows the probability distribution of the performance function G for standard variation s_d = 0.05 mm. The mean value of the performance function is m_G = 0.07849 MPa and the standard deviation is s_G = 0.04026 MPa. One may observe that part of the distribution of the performance function G remains on the positive side indicating that for s_d = 0.05 the maximum shear stress exceeds the ultimate shear stress. The area to the left of the zero (red line) indicates the probability of failure. In this case the probability that the performance function G is less than zero is 4.24 %, P[G< 0] = 4.24%.

RESULTS OF RELIABILITY BASED ANALYSIS

For various values of the standard deviation s_d the probabilistic FEA model can not only predict the mean value of the performance function m_G, the standard deviation of the performance function s_G but also the probability that the performance function is less than zero P[G < 0]. Table 2 shows Mean, Standard Deviation of Performance Function G and Probabilities of Failure for various values of standard deviation of the circularity variation δ. Figure 10 shows a plot of the probability that the performance function is less than zero versus the standard deviation of d. One may observe that for values of s_d = 0.0522, 0.0672 and 0.0818 mm the probability of failure is 5%, 10% and 15% respectively.

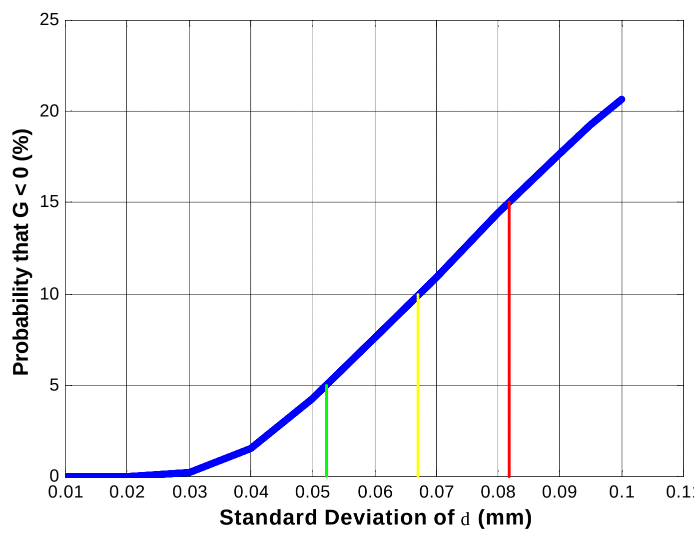

Figure 10 Probability that the Performance Function Is Less Than Zero Versus The Standard Deviation Of d

This graph can be used as a design guide to select the required standard deviation of d to achieve the desired reliability level. For example if the desired reliability level is 95% the maximum standard deviation of d should be 0.052 mm.

RESULTS OF ROBUST ANALYSIS

The robust design optimization approach not only shifts performance mean to the target value but also reduces a product's performance variability, achieving the desired **sigma level** robustness on the key product performance characteristics with respect to the quantified variation.

In this study a typical formulation of an n-sigma level robust design approach can be stated as:

Find the value of the maximum standard deviation of d in order to achieve positive values of the expression

$$m_G - n\ s_G > 0$$

Figure 11 shows a sensitivity plot of three curves for 1, 2 and 3 σ versus the standard deviation of d. For a three σ quality level the standard deviation of d cannot be more than s_δ = 0.0324 mm. For a two σ quality level the standard deviation of d cannot be more than s_δ = 0.0488 mm and for a one σ quality level the standard deviation of d cannot be more than s_δ = 0.0812.

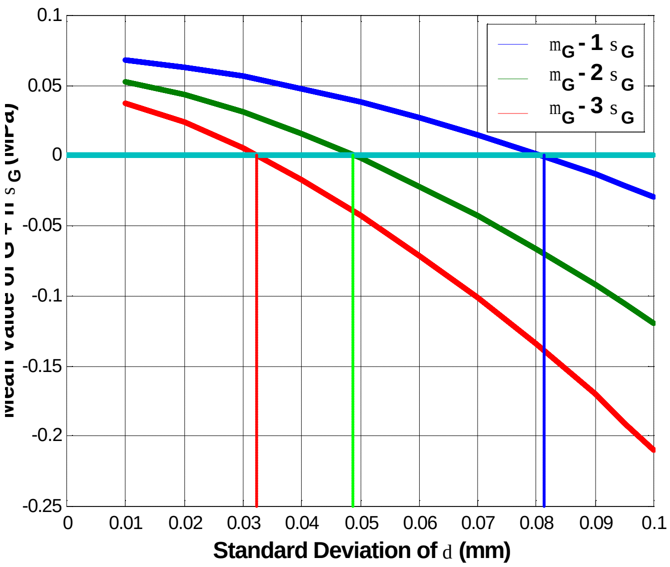

Figure 11 One, two and three σ quality level curves versus the standard deviation of δ

CONCLUSIONS

- The example presented demonstrates the advantage of using an automated probabilistic design process that enables engineers to identify better designs that meet the performance objectives and are less sensitive to manufacturing variations.

- For a given reliability goal (i.e. 95%) the maximum standard deviation of the circularity variation can be determined using the design process described. The results summary is shown in table 1. A good correlation between these results and the verification tests was found.

- For a given sigma quality level (i.e. six-sigma) the maximum standard deviation of circularity variation can be determined using the design process described. The results summary is shown in table 1.

- By incorporating the physical scatter into the model, the risk of failing legal or consumer tests can be minimized.

Table 1 Max σ_δ for various quality levels

Quality / Reliability	**Maximum** s_d (mm)
1 σ Quality Level	0.0812
2 σ Quality Level	0.0488
3 σ Quality Level	0.0324
85% Reliability Level	0.0818
90% Reliability Level	0.0672
95% Reliability Level	0.0522

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SYMBOLS

F_{max} maximum diameter of substrate

F_{min} minimum diameter of substrate

d variation in circularity (roundness) = $F_{max} - F_{min}$

m_d mean value of d

s_d standard variation of d

t_{ult} ultimate shear stress of substrate material

m_{tu} mean value of ultimate shear stress

s_{tu} standard deviation of ultimate shear stress

G performance function $G = t_{ult} - t_{max}$

m_G mean value of performance function G

s_G standard deviation of performance function G

q CC angle of a radial line with the horizontal

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Table 2 Mean, Standard Deviation of Performance Function G and Probabilities of Failure for various values standard deviation of the circularity variation δ

σ_δ (mm)	μ_G (Mpa)	σ_G (Mpa)	P[G<0] (%)	Min G (Mpa)	Max G (Mpa)
Standard Deviation of δ	Mean G Value	Standard Deviation of G	Probability that G < 0	Minimum G Value	Maximum G Value
0.01	0.08312	0.01521	0.000	0.1287	0.0342
0.02	0.08311	0.01973	0.001	-0.0053	0.1478
0.03	0.08202	0.02557	0.204	-0.0419	0.1571
0.04	0.08048	0.03254	1.562	-0.0716	0.1623
0.05	0.07849	0.04026	4.24	-0.14343	0.1637
0.06	0.07604	0.04911	7.634	-0.20243	0.1697
0.07	0.07320	0.05812	10.94	-0.26923	0.1746
0.08	0.06988	0.06807	14.41	-0.38628	0.1784
0.09	0.06612	0.07883	17.68	-0.45971	0.1830
0.095	0.06408	0.08487	19.26	-0.50724	0.1855
0.10	0.06119	0.09044	20.65	-0.54985	0.1873